Fachbereich Mathematik
Prof. Klaus Keimel
Andreas Balser

TECHNISCHE
UNIVERSITÄT
DARMSTADT

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Algebraic Topology, No. 5
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Sketches of solutions to selected problems

1. We give the results only.

   a) $\chi(S^2) = 2$

   b) Observe that the easiest triangulation of the sphere S^n is the complex of all the proper faces of an $n+1$-simplex.
   $\chi(S^{2n}) = 2$ and $\chi(S^{2n-1}) = 0$

   c) $\chi(T) = 0$

   d) $\chi(P) = 1$

   e) $\chi(K) = 0$

   f) $\chi(\underbrace{T\# \cdots \#T}_{n-\text{times}}) = -2(n-1) = 2-2n$

   g) $\chi(\underbrace{P\# \cdots \#P}_{n-\text{times}}) = 2-n$

   Additional: Show that $\chi(S_1\#S_2) = \chi(S_1) + \chi(S_2) - 2$ *for two surfaces* S_1, S_2.

   Note that the number n in parts f, g is the number which we defined as the *genus* of the surface.

2. By Exercise 1.3, a triangulation of the surface of a polyhedron is a triangulation of S^2. It is easy to check that the Euler-Poincaré characterstic of a polyhedron is equal to $F - K + E$, so the result follows from $\chi(S^2) = 2$.

3. The proof of 5.3.3. works verbatim for the case when the base ring is a field.